

New dark energy constraints from supernovae, microwave background and galaxy clustering

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Using the spectacular new high redshift supernova observations from the HST/GOODS program and previous supernova, CMB and galaxy clustering data, we make the most accurate measurements to date of the dark energy density ρ_X as a function of cosmic time, constraining it in a rather model-independent way, assuming a flat universe. We find that Einstein's vanilla scenario where $\rho_X(z)$ is constant remains consistent with these new tight constraints, and that a Big Crunch or Big Rip is more than 50 gigayears away for a broader class of models allowing such cataclysmic events. We discuss popular pitfalls and hidden priors: parametrizing the equation-of-state $w_X(z)$ assumes positive dark energy density and no Big Crunch, and the popular parametrization $w_X(z) = w_0 + w'_0 z$ has nominally strong constraints from CMB merely because $w'_0 > 0$ implies an unphysical exponential blow-up $\rho_X \propto e^{3w'_0 z}$.

The nature of dark energy has emerged as one of the deepest mysteries in physics. When strong evidence for its existence first appeared from supernova observations in 1998 [1, 2], the most pressing question was whether it was real or an observational artifact. Since then, the supernova evidence has both withstood the test of time and strengthened [3–5], and two other lines of evidence have independently led to the same conclusion: measurements of cosmological clustering with the cosmic microwave background (CMB) and large-scale structure (LSS) (e.g., [6, 7]) and observation of CMB/LSS correlations due to the late integrated Sachs-Wolfe effect [8]. Now that its current density has been accurately measured (WMAP+SDSS gives $\rho_X(0) = (4.8 \pm 1.2) \times 10^{-27} \text{ kg/m}^3$ [7], corresponding to $(9.3 \pm 2.3) \times 10^{-124}$ in Planck units and $\Omega_\Lambda \approx 0.7$), the next pressing question is clearly whether its density ρ_X stays constant over time (like Einstein's cosmological constant) or varies. The latter is predicted by most models attempting to explain dark energy either as a dynamic substance, “quintessence” (e.g., [9]), or via some form of modified gravitational theory, perhaps related to extra dimensions or string physics (e.g., [10]). See [11] for reviews with more complete lists of references.

The recent discovery of 16 Type Ia supernovae (SNe Ia) [5] with the Hubble Space Telescope during the GOODS ACS Treasury survey bears directly on this question. By discovering 6 out of the 7 highest-redshift SNe Ia known, all at $z > 1.25$, this search team [5] was able to pinpoint for the first time the transition epoch from matter domination to dark energy domination when the cosmic expansion began to accelerate. It is therefore timely to revisit this question of if and how the dark energy density varies with time. This is the goal of the present paper. Given our profound lack of understanding of dark energy and the profusion of theoretical models in the recent literature, we focus on measuring the function $\rho_X(z)$ in as model-independent a fashion as possible, emphasizing what we do and do not know given various assump-

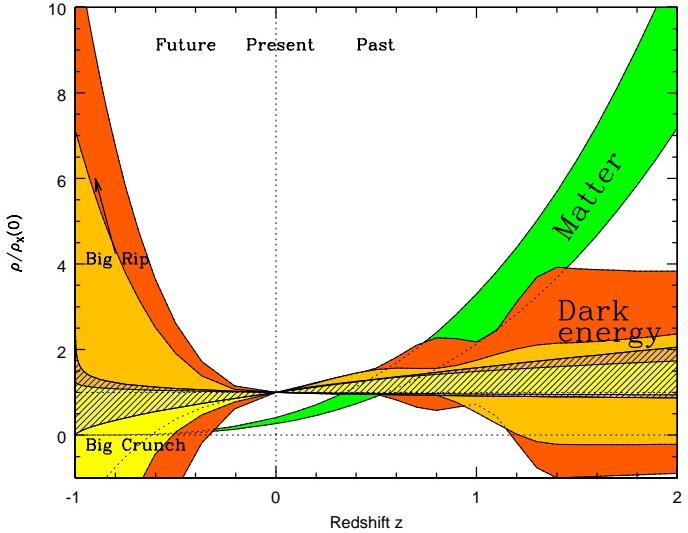


FIG. 1: 1σ constraints on the density of matter and dark energy from SN Ia (Riess sample, flux-averaged with $\Delta z = 0.05$), CMB and LSS data, all in units of the current dark energy density. From inside out, the four nested dark energy constraints are for models making increasingly strong assumptions, corresponding, respectively, to the 4-parameter spline, the 3-parameter spline, the 2-parameter (f_∞, w_i) case and the 1-parameter constant w case (hatched). The Universe starts accelerating when the total density slope $d \ln \rho / d \ln(1+z) > -2$, which roughly corresponds to when dark energy begins to dominate, i.e., to where the matter and dark energy bands cross. In the distant future, the Universe recollapses if the dark energy density ρ_X goes negative and ends in a “Big Rip” if it keeps growing ($d \ln \rho_X / d \ln(1+z) < 0$).

tions about how $\rho_X(z)$ is parametrized, about data sets used and about modeling thereof. We will see that the new data are powerful enough to make previous measurements of $\rho_X(z)$ (e.g., [12–14]) tighter and more robust and also to extend them back firmly into the epoch of cosmic deceleration.

Analysis Technique: We wish to measure the dimensionless *dark energy function*, $X(z) \equiv \rho_X(z)/\rho_X(0)$,

the dark energy density in units of its present value. We do this as described in [13], fitting to SN Ia, CMB and LSS information, obtaining the results shown in Figure 1.

The measured distance-redshift relations of SNe Ia provide the foundation for probing the dark energy function $X(z)$. In a flat Universe, the dimensionless luminosity distance $d_L(z)H_0/c = (1+z)\Gamma(z)$, where $\Gamma(z) = \int_0^z dz'/E(z')$ is the dimensionless comoving distance and

$$E(z) \equiv [\Omega_m(1+z)^3 + (1-\Omega_m)X(z)]^{1/2} \quad (1)$$

is the cosmic expansion rate relative to its present value. We use the “gold” set of 157 SNe Ia published by Riess *et al.* in [5] and analyze it using flux-averaging statistics [13, 16] to reduce bias due to weak gravitational lensing by intervening matter. We assume spatial flatness as motivated by inflation and discuss the importance of this and other assumptions below. We use CMB and LSS data to help break the degeneracy between the dark energy function $X(z)$ and Ω_m . For the CMB, we use only the measurement of the CMB shift parameter [18], $R \equiv \Omega_m^{1/2}\Gamma(z_{\text{CMB}}) = 1.716 \pm 0.062$ from CMB (WMAP, CBI, ACBAR) [6, 17], where $z_{\text{CMB}} = 1089$. The only large-scale structure information we use is the linear growth rate $f(z_{\text{2df}}) = 0.51 \pm 0.11$ measured by the 2dF galaxy redshift survey (2dFGRS) [3, 19], where $z_{\text{2df}} = 0.15$ is the effective redshift of this survey and $f \equiv (d \ln D / d \ln a)$ is determined by solving the equation for the linear growth rate D , $D''(\tau) + 2E(z)D'(\tau) - \frac{3}{2}\Omega_m(1+z)^3D = 0$, where primes denote $d/d(H_0 t)$. Note that the CMB and LSS measurements we use (R and f) do *not* depend on the Hubble parameter H_0 , and are quite insensitive to assumptions made about $X(z)$. The SN Ia measurements used are also independent of H_0 , since we marginalize them over the intrinsic SN Ia luminosity calibration.

We run a Monte Carlo Markov Chain (MCMC) based on the MCMC engine of [20] to obtain a few million samples of Ω_m and $X(z)$. The dark energy bands in Figure 1 correspond to the central 68% of the X -values at each z and the matter band does the same for $\rho_m(z)/\rho_X(0) = (1+z)^3\Omega_m/(1-\Omega_m)$.

Results: Figure 1 shows our main results, the constraints on the dark energy function $X(z) = \rho_X(z)/\rho_X(0)$ for four different parametrizations, and illustrates that the assumptions one makes about the curve $X(z)$ have an important effect on the results. The most common way of measuring dark energy properties in the literature has been to parametrize the dark energy function X by merely one or two free parameters, constraining these by fitting to observed data. Table 1 includes the historically most popular parametrizations, expressed as functions of the dimensionless cosmic scale factor $a \equiv (1+z)^{-1}$. Parametrization A simply assumes that $X(a)$ is a power law, with the single equation-of-state parameter w determining its logarithmic slope. From the identity $\partial \ln \rho_X / \partial \ln a = -3(1+w_x)$, it follows that parametrization B corresponds to the popular

parametrization $w_x(z) = w_0 + w'_0 z$ [26], which has been widely used in the literature. It has the drawback of being rather unphysical for $w'_0 > 0$, with the dark energy density $\rho_X(z)$ blowing up as $e^{3w'_0 z}$ at high redshift. Parametrization C avoids this [21], and corresponds to $w_x = w_1 + w_a(1-a)$, but blows up exponentially in the future as $a \rightarrow \infty$ for $w_a > 0$. In contrast, our parametrization D remains well-behaved at all times: both early on and in the distant future, the dark energy approaches either a constant equation of state w_i or a constant density, depending on the sign of $(1+w_i)$.

Obviously, the more restrictive the assumptions about X are, the stronger the nominal constraints will be, so it is crucial to be clear on what these assumptions are. For instance, Table 1 shows that parametrizations A, B and C all tacitly assume that $X(z) \geq 0$, *i.e.*, that the dark energy density cannot be negative, hence ruling out by fiat the possibility that the Universe can recollapse in a Big Crunch. Note that even *arbitrary* function $w(z)$ has this hidden assumption built in.

To introduce as little theoretical bias as possible into our measurement, we use parametrizations E and F from Table 1; these are fairly model-independent reconstructions of the dark energy function $X(z)$, assuming merely that $X(z)$ is a sufficiently smooth function that it can be modeled with a cubic spline out to some redshift z_{max} , and by a constant- w power law thereafter. We choose z_{max} to avoid sparse SN Ia data, and parametrize X by its values at N equispaced spline points at $z_{\text{max}}/N, 2z_{\text{max}}/N, \dots, z_{\text{max}}$. $X(z)$ is matched smoothly on to $(1+z)^{3(1+w_i)}$ at $z > z_{\text{max}}$. This specifies $X(z)$ uniquely once we require $X(z)$ and $X'(z)$ to be everywhere continuous and set $X(0) = 1$, $X'(0) = X(z_1)/z_1$. We have chosen $z_{\text{max}} = 1.4$, as there are only two SNe Ia at higher redshifts. Since $X(z)$ is only very weakly constrained beyond $z > z_{\text{max}}$, we impose a prior of $w_i \geq -2$ to avoid an unbounded parameter space. Changing the prior to $w_i \geq -20$ or changing the functional form of $X(z)$ at $z > z_{\text{max}}$ (to an exponential, for example) has little impact on the reconstructed $X(z)$. We also find our results to be rather robust to data details. Including the “silver” sample from [5] does not change our results qualitatively, and replacing the CMB shift parameter we used ($R = 1.716 \pm 0.062$) by $R = 1.710 \pm 0.137$ (from WMAP data alone [6]) broadens the 68% confidence envelope by less than 20%.

Figure 1 also shows the constraints on the dark energy function $X(z)$ corresponding to parametrizations A and D from Table 1, imposing the priors $w_i \geq -2$ and $f_\infty \geq 0$ for D. For comparison with the results of [5], we also studied parametrization B, with a weak prior $w'_0 \geq -20$ to avoid an unbounded parameter space. Note that MCMC tacitly assumes uniform prior on the parameters, so if the parameter space is unbounded, the MCMC will drift off in the unbounded direction and never converge. Reparametrizing changes this implicit prior by the Jacobian of the transformation. Although we have imposed minimal priors to avoid unbounded parameter

Table 1: Parametrizations used for the dark energy function $X \equiv \rho_X(z)/\rho_X(0)$ in terms of the cosmic scale factor $a = (1+z)^{-1}$.

Parametrization	n Parameters	Definition
A) Constant eq. of state w	1 w	$X = a^{-3(1+w)}$
B) Affine $w(z)$	2 w_0, w'_0	$X = a^{-3(1+w_0-w'_0)} e^{3w'_0(a^{-1}-1)}$
C) Affine $w(a)$	2 w_1, w_a	$X = a^{-3(1+w_1+w_a)} e^{3w_a(a-1)}$
D) Forever regular	2 w_i, f_∞	$X = f_\infty + (1-f_\infty)a^{-3(1+w_i)}$
E) 3-parameter spline	3 $w_i, X(z_1), X(z_2)$	Cubic spline in z for $z \leq z_2$, $X = X(z_2) \left(\frac{1+z}{1+z_2}\right)^{3(1+w_i)}$ for $z \geq z_2$
F) 4-parameter spline	4 $w_i, X(z_1), X(z_2), X(z_3)$	Cubic spline in z for $z \leq z_3$, $X = X(z_3) \left(\frac{1+z}{1+z_3}\right)^{3(1+w_i)}$ for $z \geq z_3$

space where $X(z)$ can be arbitrarily close to zero, but we have *not* imposed priors motivated by any theoretical model. For example, scalar-field models typically have $X'(a) \leq 0$, since fields usually roll down potentials, not up. In addition, many models prohibit the dark energy density from being negative. However, we do not wish to assume such priors, since “dark energy” could be a manifestation of something completely different, like modified gravity [10].

As has been emphasized [22–24], SN Ia data are sensitive only to the smooth, overall shape of $X(z)$. This is because the error bars on sharp features on a scale Δz are proportional to $(\Delta z)^{-3/2}$ due to the derivative involved in going from comoving distance $r(z)$ to dark energy function X [23] — reconstructing $w_X(z)$ is still harder, the requirement that one effectively take the second derivative of noisy data [14] giving the error scaling as $(\Delta z)^{-5/2}$ [23]. Figure 1 shows that as we allow more small-scale freedom by parametrizing $X(z)$ by 1, 2, 3 and 4 parameters, the allowed bands become thicker. However, the broader bands generally encompass the narrower ones, showing no hint in the data that the true $X(z)$ has funny features outside of the 1- and 2-parameter model families. Indeed, all bands are seen to be consistent with the simplest model of all: the zero-parameter “vanilla” model $X(z) = 1$ corresponding to Einstein’s cosmological constant.

In other words, faced with the fact that an analysis using parametrization A implies $w \approx -1$ (we obtain $w = -0.91_{-0.15}^{+0.13}$ combining SN Ia, CMB and LSS), readers hoping for something more interesting than vanilla may correctly argue that these constraints are dominated by accurate measurements at lower redshift and may fail to reveal hints of an upturn in $X(z)$ at $z \gtrsim 1$ because parametrization A incorrectly assumes that $(\log a, \log X)$ is a straight line. Our more general parametrizations close this loophole by allowing $X(z)$ much greater freedom, and the fact that none of them provide any hint yet of non-vanilla dark energy behavior therefore substantially strengthens the case for a simple cosmological constant, $X(z) = 1$.

What is the ultimate fate of the Universe? If for any of our models ρ_X eventually goes negative so that total density drops to zero at some time t_{turn} , then the expansion reverses and a Big Crunch occurs at $t = 2t_{\text{turn}}$ — this applies only if X is uniquely determined by the cosmic scale factor (equivalently z) as in Table 1, and not for many scalar field models [27]. The cosmic time

$t = \int da/\dot{a} = \int H^{-1} d\ln a$, and if this asymptotes to a finite value as $a \rightarrow \infty$, then a cataclysmic Big Rip [15] occurs at this time. This is equivalent to $w(z) < -1$ at $z = -1$, so parametrizations A, B and C rip if $w < -1$, $w_0 - w'_0 < -1$ and $w_a > 0$, respectively.

Predictions for the future need to be taken with a large grain of salt, since they are obviously highly model-dependent. For instance, parametrizations A, B and C cannot crunch, whereas E and F cannot rip. Simply combining all MCMC models from all our parametrizations, we find that 95% of them last at least another 49 gigayears, 25% ending in a Big Crunch, 8% ending in a Big Rip and 67% quietly expanding forever.

Caveats and potential pitfalls: When interpreting dark energy constraints such as those that we have presented, two crucial caveats must be borne in mind: potential SN Ia systematic errors and potential false assumption about other physics. We refer the reader to [3, 5] for thorough discussions of the former and focus on the latter.

The SN Ia, CMB and LSS measurements we have used involve only $X(z)$, Ω_m and Ω_{tot} . Because of degeneracies between these three quantities, the inferences about $X(z)$ therefore depend strongly on the assumptions about the two cosmological parameters Ω_m and Ω_{tot} . Yet it is all too common to constrain dark energy properties using prior information about Ω_m and Ω_{tot} that in turn hinges on assumptions about the dark energy, usually the vanilla assumption $X(z) = 1$, a pitfall emphasized by, *e.g.*, [24].

We have assumed flat space, $\Omega_{\text{tot}} = 1$, as have virtually all recent publications measuring dark energy properties (usually using parametrizations A, B or C). It is well-known that this assumption is crucial: introducing Ω_{tot} as a free parameter to be marginalized over has such a dramatic effect on luminosity distances that essentially no interesting constraints can be placed on $X(z)$ at the present time, not even assuming the highly restrictive parametrization A. We will present a detailed investigation of dark energy independent constraints on Ω_{tot} from CMB and LSS elsewhere.

We now turn to the issue of dark-energy independent constraints on Ω_m . As emphasized by [24], assumptions about Ω_m make a crucial difference as well. As an example, Figure 2 shows the constraints on (w_0, w'_0) for parametrization B. The left panel illustrates that the constraints from SN Ia alone are much weaker than those obtained by imposing a strong prior $\Omega_m = 0.27 \pm 0.04$ as was done in Figure 10 of [5]. Although this prior co-

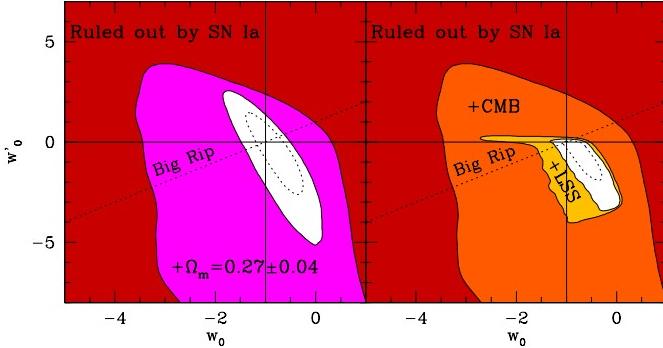


FIG. 2: How constraints on w_0 and w'_0 depend on assumptions and data used. Darker shaded regions are ruled out at 95% confidence by SNe Ia alone; lighter shaded regions are ruled out when adding other information as indicated. 68% contours are dotted. Models above the dotted line end in a Big Rip. The 157 SNe Ia (Riess sample) have been flux-averaged with $\Delta z = 0.05$.

incides with the measurement of Ω_m from WMAP and 2dFGRS [6], it should *not* be used here since it assumes $X(z) = 1$. The right panel of Figure 2 shows the effect of including CMB information self-consistently (via the R -parameter) in our constraints. We see that w_0 -values as low as -3 remain allowed, as expected given the above-mentioned weak Ω_m -constraints, and that additional information (in this case from LSS) is needed to tighten things up. This panel also illustrates the hazard of poor dark energy parametrizations: the seemingly impressive upper limit on w'_0 tells us nothing whatsoever about dark energy properties via SN Ia, but merely reflects that the unphysical exponential blowup $X \propto e^{3w'_0 z}$

would violate the CMB constraint.

Conclusions: In conclusion, we have reported the most accurate measurements to date of the dark energy density ρ_X as a function of time, assuming a flat universe. We have found that in spite of their constraining power, the spectacular new high- z supernova measurements of [5] provide no hints of departures from the vanilla model corresponding to Einstein's cosmological constant. This is good news in the sense of simplifying the rest of cosmology, but dims the prospects that nature will give us quantitative clues about the true nature of dark energy by revealing non-vanilla behavior. The apparent constancy of $\rho_X(z)$ also makes attempts to explain away dark energy by blaming systematic errors appear increasingly contrived, further strengthening the evidence that dark energy is real and hence a worthy subject of study. Future experiments [25] can dramatically shrink the error bars in Figure 1, and therefore hold great promise for illuminating the nature of dark energy.

Public software: A Fortran code that uses flux-averaging statistics to compute the likelihood of an arbitrary dark energy model (given the SN Ia data from [5]) can be found at <http://www.nhn.ou.edu/~wang/SNcode/>.

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